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Measure of classical correlation in a two-qubit state

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Abstract
We suggest a measure of classical correlation in a two-qubit state, which is computable and implementable in experiments. As examples, we investigate the classical and quantum correlations in the Werner states and in a group of particular states mixed with three parts: an entangled, a separable and a product one. It is shown that the quantum correlation in a two-qubit state cannot exceed the classical correlation in the same state.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Early discussion on the correlation between two particles in a quantum domain can be traced back to the EPR paradox [1], Bell's theorem [2, 3] and the corresponding experimental tests [4–6]. How to quantify the classical and quantum correlation in a bipartite state is always an interesting question. During the last decade, many good suggestions have been proposed to measure the degree of entanglement [7–12]. At the same time, the quantification of classical correlation in composite states also attracted much attention [10, 13–16]. For example, Henderson and Vedral defined the classical correlation in a quantum state as the maximum difference in Von Neumann entropy (uncertainty) about one subsystem before and after some positive-operator-valued measures (POVMs) on the other subsystem [14]; Groisman *et al* gave an operational definition for the quantum and classical correlation in a bipartite quantum state via the amount of work (noise) required to destroy the corresponding correlations [16]. However, the permutation symmetry of the measure proposed in [14] is not confirmed; how to find out a POVM, which optimizes this classical correlation in a general case, is not obvious [17]. Quantifying the 'work' or 'noise' in [16] is not an easy task, too. At present, although there is no consensus on the quantification of classical correlation in a quantum state, some necessary properties are still expected for an acceptable one. In the following, we use the notation $\mathrm{R}(\rho_{AB})$ to represent the classical correlation in an A–B two-qubit state ρ_{AB}, which should satisfy

(1) $\mathrm{R}(\rho_{AB}) = 0$ iff ρ_{AB} is a product state,
(2) $\mathrm{R}(\rho_{AB})$ is symmetric under the permutation of A and B,
(3) $\mathrm{R}(\rho_{AB})$ is invariant under local unitary operations,
(4) $\mathrm{R}(\rho_{AB})$ equals the quantum correlation (entanglement) $\mathrm{E}(\rho_{AB})$ if the two-qubit state ρ_{AB} is a pure state.

2. Definition of classical correlation

Our definition of classical correlation is based on the results of local projective measurements. Given two local projections $\hat{M}_A$ and $\hat{M}_B$ on a two-qubit state ρ_{AB}, we first introduce an un-normalized correlation function through the covariance of the two local projections, which is

$$\begin{aligned} f(\rho_{AB}) &\equiv \langle(\hat{M}_A - \langle\hat{M}_A\rangle)(\hat{M}_B - \langle\hat{M}_B\rangle)\rangle_{AB} \\ &= \langle\hat{M}_A \otimes \hat{M}_B\rangle_{AB} - \langle\hat{M}_A\rangle_A\langle\hat{M}_B\rangle_B. \end{aligned} \tag{1}$$

Here $\langle\hat{M}_A \otimes \hat{M}_B\rangle_{AB}$ stands for the expectation value of the coincidence measurement $\hat{M}_{AB} = \hat{M}_A \otimes \hat{M}_B$ on the A–B composite system, while $\langle\hat{M}_{A(B)}\rangle_{A(B)}$ represents the expectation value of the local measurement $\hat{M}_{A(B)}$ on the subsystem A (B). By applying the Cauchy–Schwarz inequality, it is easy to get the upper bound of this un-normalized correlation function, i.e. $f(\rho_{AB}) \leqslant \frac{1}{4}$. Thus, the function (1) can be normalized (with maximum value 1) as

$$\mathrm{F}(\rho_{AB}) = 4f(\rho_{AB}) = 4(\langle\hat{M}_A \otimes \hat{M}_B\rangle_{AB} - \langle\hat{M}_A\rangle_A\langle\hat{M}_B\rangle_B), \tag{2}$$

whose absolute value indicates the correlation strength between qubits A and B. The lower bound '-1' and the upper bound '1' correspond to perfect anti-correlation and perfect correlation, respectively.

Based on the correlation function (2), a pair of local measurements, $\hat{M}_A$ and $\hat{M}_B$, give a measure of the correlation function in the state ρ_{AB}. However, this measure may not be the optimal one if the two local projections, $\hat{M}_A$ and $\hat{M}_B$, are chosen randomly. In experiments, if we scan all pairs of local projections, a maximum correlation result can be obtained. Here we define the classical correlation in a two-qubit state ρ_{AB} as the maximum value of the normalized correlation function (2), as the two local measurements, $\hat{M}_A$ and $\hat{M}_B$, run throughout all local projections, that is

$$\mathrm{R}(\rho_{AB}) = \max_{\hat{M}_A \in M_A;\, \hat{M}_B \in M_B} 4(\langle\hat{M}_A \otimes \hat{M}_B\rangle_{AB} - \langle\hat{M}_A\rangle_A\langle\hat{M}_B\rangle_B), \tag{3}$$

where M_A (and M_B) is the set of all projective measurements in the subsystem A (and B). Since all projective measurements in each subsystem are unitarily equivalent to each other, the projection $\hat{M}_j$ ($j = A, B$) in a qubit system can be described by a specific projection $\hat{P}_j$ and a unitary matrix $\hat{U}_j$ through the relation $\hat{M}_j = \hat{U}_j^\dagger \hat{P}_j \hat{U}_j$, with

$$\hat{P}_j = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_j, \tag{4a}$$

$$\hat{U}_j = \begin{pmatrix} \cos\theta_j\, \mathrm{e}^{\mathrm{i}\phi_j} & \sin\theta_j \\ -\sin\theta_j\, \mathrm{e}^{\mathrm{i}\phi_j} & \cos\theta_j \end{pmatrix}_j \tag{4b}$$

and

$$\hat{M}_j = \begin{pmatrix} \cos^2\theta_j & \sin\theta_j\cos\theta_j\, \mathrm{e}^{-\mathrm{i}\phi_j} \\ \sin\theta_j\cos\theta_j\, \mathrm{e}^{\mathrm{i}\phi_j} & \sin^2\theta_j \end{pmatrix}_j. \tag{4c}$$

Noting that the unitary matrix (4b) is equivalent to the combined action of a phase shifter and a beam merger, i.e. $\hat{U}_j = \hat{S}_j^{(2)}\hat{S}_j^{(1)}$, with $\hat{S}_j^{(1)} = \exp\left(\mathrm{i}\frac{\phi_j}{2}\sigma_z\right)$ and $\hat{S}_j^{(2)} = \exp(\mathrm{i}\theta_j\sigma_y)$ [18–20], the classical correlation defined in equation (3) is experimentally achievable in a two-particle four-path interferometer [21–23].

3. Measure of classical correlation

By applying the two local projections, $\hat{M}_A$ and $\hat{M}_B$, with the form of equation (4*c*), on the *A*–*B* two-qubit state ρ_{AB}, we can derive the three expectation values involved in equation (3), which are

$$\langle\hat{M}_A\rangle_A = (\rho_{11}+\rho_{22})\cos^2\theta_A + (\rho_{33}+\rho_{44})\sin^2\theta_A + \mathrm{Re}[(\rho_{13}+\rho_{24})\sin 2\theta_A\,\mathrm{e}^{\mathrm{i}\phi_A}], \tag{5a}$$

$$\langle\hat{M}_B\rangle_B = (\rho_{11}+\rho_{33})\cos^2\theta_B + (\rho_{22}+\rho_{44})\sin^2\theta_B + \mathrm{Re}[(\rho_{12}+\rho_{34})\sin 2\theta_B\,\mathrm{e}^{\mathrm{i}\phi_B}], \tag{5b}$$

$$\begin{aligned}\langle\hat{M}_A\otimes\hat{M}_B\rangle_{AB} = {}& \rho_{11}\cos^2\theta_A\cos^2\theta_B + \rho_{22}\cos^2\theta_A\sin^2\theta_B + \rho_{33}\sin^2\theta_A\cos^2\theta_B \\ &+ \rho_{44}\sin^2\theta_A\sin^2\theta_B + \mathrm{Re}[\rho_{13}\sin 2\theta_A\cos^2\theta_B\,\mathrm{e}^{\mathrm{i}\phi_A} + \rho_{12}\cos^2\theta_A\sin 2\theta_B\,\mathrm{e}^{\mathrm{i}\phi_B} \\ &+ \tfrac{1}{2}\rho_{14}\sin 2\theta_A\sin 2\theta_B\,\mathrm{e}^{\mathrm{i}(\phi_A+\phi_B)} + \tfrac{1}{2}\rho_{23}\sin 2\theta_A\sin 2\theta_B\,\mathrm{e}^{\mathrm{i}(\phi_A-\phi_B)} \\ &+ \rho_{24}\sin 2\theta_A\sin^2\theta_B\,\mathrm{e}^{\mathrm{i}\phi_A} + \rho_{34}\sin^2\theta_A\sin 2\theta_B\,\mathrm{e}^{\mathrm{i}\phi_B}].\end{aligned} \tag{5c}$$

Here $\rho_{mn}(m,n = 1,2,3,4)$ are the elements of the density matrix ρ_{AB}. By substituting equations (5) into equation (3), we can rewrite the classical correlation in the two-qubit state ρ_{AB} as

$$\begin{aligned}\mathrm{R}(\rho_{AB}) = \max_{\theta_A,\theta_B,\phi_A,\phi_B} & 4(a_1\cos 2\theta_A\cos 2\theta_B + a_2\sin 2\theta_A\cos 2\theta_B \\ &+ a_3\cos 2\theta_A\sin 2\theta_B + a_4\sin 2\theta_A\sin 2\theta_B),\end{aligned} \tag{6a}$$

with

$$\begin{aligned}a_1 &= \rho_{11}\rho_{44} - \rho_{22}\rho_{33}, \\ a_2 &= \mathrm{Re}\{[(\rho_{22}+\rho_{44})\rho_{31} - (\rho_{11}+\rho_{33})\rho_{42}]\,\mathrm{e}^{\mathrm{i}\phi_A}\}, \\ a_3 &= \mathrm{Re}\{[(\rho_{33}+\rho_{44})\rho_{21} - (\rho_{11}+\rho_{22})\rho_{43}]\,\mathrm{e}^{\mathrm{i}\phi_B}\}, \\ a_4 &= \tfrac{1}{2}\mathrm{Re}\{[\rho_{32} - (\rho_{31}+\rho_{42})(\rho_{12}+\rho_{34})]\,\mathrm{e}^{\mathrm{i}(\phi_A-\phi_B)} + [\rho_{41} - (\rho_{31}+\rho_{42})(\rho_{21}+\rho_{43})]\mathrm{e}^{\mathrm{i}(\phi_A+\phi_B)}\}.\end{aligned} \tag{6b}$$

Since the four coefficients, a_1, a_2, a_3 and a_4, are independent of the two variables θ_A and θ_B, we can first maximize the function (6) against θ_A and θ_B, which leads to

$$\mathrm{R}(\rho_{AB}) = \max_{\phi_A,\phi_B} 2\left[\sqrt{(a_1+a_4)^2 + (a_2-a_3)^2} + \sqrt{(a_1-a_4)^2 + (a_2+a_3)^2}\right]. \tag{7}$$

Although the analytic solution for the maximization of the function (7) with respect to two angle variables, ϕ_A and ϕ_B, is not evident, its numerical result can be easily computed. Thus, based on the results of local projective measurements, we have derived a dimensionless quantity to measure the classical correlation in a two-qubit state, which is computable and experimentally implementable. Furthermore, it is easy to verify that the four properties listed in section 1 are fully satisfied by this measure of classical correlation.

According to the above result (7), if a two-qubit state $\tilde{\rho}_{AB}$ possesses the following 'symmetries',

$$(\tilde{\rho}_{11}+\tilde{\rho}_{22})\tilde{\rho}_{34} = (\tilde{\rho}_{33}+\tilde{\rho}_{44})\tilde{\rho}_{12}, \qquad (\tilde{\rho}_{11}+\tilde{\rho}_{33})\tilde{\rho}_{24} = (\tilde{\rho}_{22}+\tilde{\rho}_{44})\tilde{\rho}_{13}, \tag{8}$$

the corresponding classical correlation has a simpler form,

$$\mathrm{R}(\tilde{\rho}_{AB}) = \max\{T_1, T_2\}, \tag{9a}$$

with

$$T_1 = 4|\tilde{\rho}_{11}\tilde{\rho}_{44} - \tilde{\rho}_{22}\tilde{\rho}_{33}|, \tag{9b}$$

$$T_2 = 2|\tilde{\rho}_{41} - (\tilde{\rho}_{31} + \tilde{\rho}_{42})(\tilde{\rho}_{21} + \tilde{\rho}_{43})| + 2|\tilde{\rho}_{32} - (\tilde{\rho}_{31} + \tilde{\rho}_{42})(\tilde{\rho}_{12} + \tilde{\rho}_{34})|. \tag{9c}$$

All two-qubit Werner states [25] and the Schmidt decomposition of any two-qubit pure state satisfy the properties (8), so the classical correlation in them can be calculated directly through equations (9).

4. Discussions and results

To quantify the correlation strength between two and more particles, different strategies may lead to different results and even different units. For example, distillable entanglement, entanglement cost, relative entropy of entanglement and entanglement of formation have the same unit as entropy. Although all of them reduce to the Von Neumann entropy for pure states, they usually present different results when the entanglement of a mixed state is requested. Concurrence and negativity, which could be considered as dimensionless measures for the entanglement of a two-qubit state, usually do not match each other unless some special states are considered. As two of the most discussed entanglement measures for two-qubit systems, entanglement of formation E_F is connected to concurrence C through the following relation [7, 11]:

$$E_F(\rho_{AB}) = h\left(\frac{1+\sqrt{1-C^2}}{2}\right), \tag{10}$$

with $h(x)$ being the binary entropy function:

$$h(x) = -x\log_2 x - (1-x)\log_2(1-x). \tag{11}$$

Accordingly, we assume that the entropic measure of the classical correlation in a two-qubit state, denoted as $\mathrm{R_E}(\rho_{AB})$, has the same one-to-one correspondence with the dimensionless measure of classical correlation $\mathrm{R}(\rho_{AB})$, that is

$$\mathrm{R_E}(\rho_{AB}) = h\left(\frac{1+\sqrt{1-\mathrm{R}^2}}{2}\right). \tag{12}$$

This definition assures the prediction that a bipartite pure state has an equal amount of the classical and quantum correlation, or say, an equal amount of noise (work) is required to erase the classical and quantum correlation in a bipartite pure state [16]. In addition, the entanglement of formation of a two-qubit state can also be considered as the minimum number of singlets required to construct per copy of the original two-qubit state, by means of local operations and classical communications (LOCC). An open question arises: whether this entropic measure for the classical correlation in a two-qubit state, defined in equation (12), has a similar meaning with respect to local unitary operations?

Without proof, most people hold the conjecture that the quantum correlation cannot exceed the classical one [16]. To test this conjecture numerically, we randomly choose one million two-qubit states in a FORTRAN program, and no violation is found based on the entanglement of formation (10) for quantum correlation and the measure (12) for the classical correlation. Since the entanglement of formation is an upper bound for all entropic measures of two-qubit entanglement [24], the above results also hold when other entanglement measures, such as distillable entanglement and relative entropy of entanglement, are chosen to represent

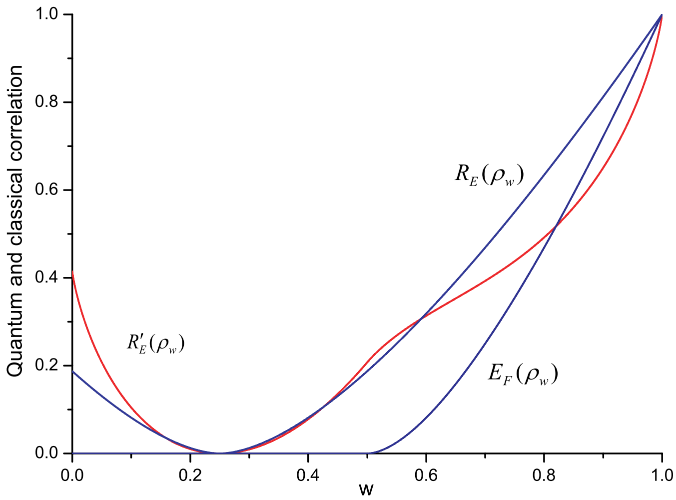

Figure 1. Quantum and classical correlation in Werner states as a function of variable w.

the quantum correlation. Another important advantage of entanglement of formation is its computability.

In the following, we present the results of quantum and classical correlation in some special states. As is well known, the Werner state [25] of this form

$$\rho_w = \frac{1-w}{3} I_4 + \frac{4w-1}{3} |\Psi^-\rangle\langle\Psi^-|, \tag{13}$$

is entangled for $\frac{1}{2} < w \leqslant 1$, with concurrence $C(\rho_w) = 2w - 1$ ($|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ is a Bell state). According to definition (3), the classical correlation in the above Werner states is

$$\mathrm{R}(\rho_w) = \left|\frac{4w-1}{3}\right|, \tag{14}$$

and the corresponding entropic measure can be derived through equation (12):

$$\mathrm{R_E}(\rho_w) = h\left(\frac{1+\sqrt{1-\mathrm{R}^2}}{2}\right) = h\left(\frac{1}{2} + \frac{1}{3}\sqrt{(1+2w)(2-2w)}\right). \tag{15}$$

In figure 1, we plot the classical correlation $\mathrm{R_E}(\rho_w)$, together with the quantum correlation (entanglement of formation) $E_\mathrm{F}(\rho_w)$, as a function of the variable w. It is shown that the classical correlation is always greater than or equal to the quantum correlation, which is in accord with the conjecture mentioned above. The only zero value on the curve $\mathrm{R_E}(\rho_w)$ for the classical correlation appears at the point with $w = 0.25$, which corresponds to the product state $I_4/4$. For $w = 1$, corresponding to the pure state $|\Psi^-\rangle$, the classical correlation equals the quantum correlation.

Just as mentioned before, some difficulties, such as the determination of POVM optimizing the classical correlation proposed in [14] and the quantification of the 'work' or 'noise' suggested in [16], make the classical correlation proposed in [10, 13–16] hard to compute. In two recent works [26, 27], the classical correlation, denoted by $\mathrm{R'_E}$ here, is also defined as the difference between the quantum mutual information and the entropic quantum correlation, which is numerically computable for the two-qubit states. Since the four properties introduced above are also satisfied by the measure in [26, 27], we would like to compare it with the quantum correlation. Based on this measure, the entropic classical correlation in the Werner states (13) would be

$$\mathrm{R}'_{\mathrm{E}}(\rho_w) = \begin{cases} 2 + w\log_2 w + (1-w)\log_2\left(\frac{1-w}{3}\right) & \text{when } 0 \leqslant w \leqslant \frac{1}{2} \\ 2 + w\log_2 w + (1-w)\log_2\left(\frac{1-w}{3}\right) + \left[\frac{1}{2} + \sqrt{w(1-w)}\right] & \\ \quad \times \log_2\left[\frac{1}{2} + \sqrt{w(1-w)}\right] + \left[\frac{1}{2} - \sqrt{w(1-w)}\right]\log_2 & \\ \quad \times \left[\frac{1}{2} - \sqrt{w(1-w)}\right] & \text{when } \frac{1}{2} < w \leqslant 1, \end{cases} \tag{16}$$

which is plotted in figure 1 ($\mathrm{R}'_{\mathrm{E}}(\rho_w)$). Different from our measure (3), the classical correlation calculated with equation (16) may be smaller than the quantum correlation (see $w > 0.82$ in figure 1) and larger than the quantum correlation (see $w < 0.82$). According to Bell's theorem [2, 3] and its experimental tests [4–6], the violation of Bell's inequality is an evidence that quantum correlation is stronger than the classical correlation. To this point of view, the quantity to measure the classical correlation in a quantum state needs to be always larger (including equal) than the quantum correlation or be always smaller (including equal) than the quantum correlation. Therefore, the measure of the classical correlation defined in [26, 27] might not be a good one.

To gain more insight on the classical and the quantum correlation in the quantum states, we present another example by constructing the following state:

$$\rho' = p|\phi^+\rangle\langle\phi^+| + q(1-p)\rho^{(s)} + (1-q)(1-p)I_4/4, \tag{17}$$

where $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is one of the four Bell states with maximal quantum correlation and classical correlation,

$$\rho^{(s)} = \begin{pmatrix} 0.3000 \\ 00.300 \\ 000.10 \\ 0000.3 \end{pmatrix}$$

is a separable state with classical correlation $\mathrm{R}(\rho^{(s)}) = 0.24$ and $I_4/4$ is a product state with vanishing quantum correlation and vanishing classical correlation. As a function of the two variables p and q, the classical correlation in the states (17) based on our measure (3) is expressed as

$$\mathrm{R}(\rho') = \tfrac{1}{5}(5p + q - pq) + \tfrac{1}{25}q^2(1-p)^2. \tag{18}$$

Note that the classical correlation is unit for the Bell state $|\phi^+\rangle$, 0.24 for the separable state $\rho^{(s)}$ and zero for the product state $I_4/4$. Note that the above result (18) is different from the average classical correlation of the three components in (17). To calculate the concurrence of the state (17), we need to know the four eigenvalues of the matrix $\rho'\tilde{\rho}' = \rho'(\sigma_y \otimes \sigma_y)\rho'^*(\sigma_y \otimes \sigma_y)$ [7, 11], which are

$$\begin{aligned} \lambda_1 &= \left[p + \frac{1-p}{20}(5+q)\right]^2, \\ \lambda_2 &= \left(\frac{1-p}{20}\right)^2(5+q)^2, \\ \lambda_3 = \lambda_4 &= \left(\frac{1-p}{20}\right)^2(5+q)(5-3q). \end{aligned} \tag{19}$$

Then the concurrence of the state (17) is finally expressed as

$$\begin{aligned} C(\rho') &= \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\} \\ &= \max\left\{0, p - \frac{1-p}{10}\sqrt{(5+q)(5-3q)}\right\}. \end{aligned} \tag{20}$$

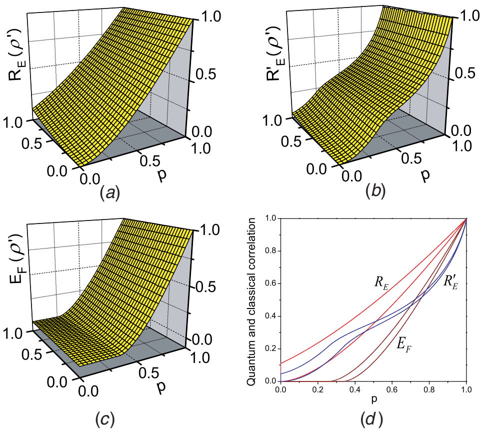

Figure 2. Classical (a and b) and quantum (c) correlation in the states (17) as a function of variables p and q. The classical correlation in graph (a) is based on our present measure and the classical correlation in graph (b) is based on the measure of [27, 26]. The graph (d) is the projection of the three graphs onto the x–z (p-correlation) plane.

From formulae (18) and (20), the inequality $\mathrm{R}(\rho') \geqslant p \geqslant C(\rho')$ holds for any p and q in the range [0, 1], which means that the quantum correlation does not exceed the classical one based on our measure. The state (17) is separable if the weight of the component $|\phi^+\rangle$ is small $\left(p < \frac{\sqrt{3}}{5+\sqrt{3}}\right)$, which implies that the concurrence is not additive for mixed states and the mixture of an entangled state with one or more separable states may result in an unentangled state. The dependence of the (entropic) classical and quantum correlation in the state (17) on the parameters p and q is plotted in figures 2(a) and (c), respectively. Figure 2(b) represents the classical correlation by the method of [26, 27], which may be larger or smaller than the quantum correlation (see figure 2(d)).

The current measure of classical correlation in a two-qubit state can be extended to high dimensional bipartite systems. By scanning all pairs of local projections in high dimensions, we can, in principle, experimentally measure the classical correlation in a high dimensional bipartite state. Unfortunately, due to the complexity of high dimensional space, we have not achieved a computable function for the quantification of the classical correlation in high dimensional states at present. On the other hand, whether this measure can be generalized to the multipartite case is still not clear. Recently, Kaszlikowski *et al* claimed that genuine quantum correlations can exist in some multipartite states which have no genuine multipartite classical correlations [28], which is different from the result of bipartite case discussed in this paper. What is the physics behind this difference is a very interesting question.

5. Conclusions

In this paper, the classical correlation in a two-qubit state is investigated, and a computable measure on the classical correlation is proposed, which can be implemented in experiments. Our numerical results show that the quantum correlation cannot exceed the classical correlation. As examples, the classical correlation in the Werner states and in a group of particular states mixed with three parts, an entangled, a separable and a product state, is presented. We hope all these discussions are helpful in understanding the classical correlation in a quantum state and its relationship with entanglement.

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References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[2] Bell J S 1964 *Physics* **1** 195
[3] Bell J S 1966 *Rev. Mod. Phys.* **38** 447
[4] Freedman S J and Clauser J F 1972 *Phys. Rev. Lett.* **28** 938
[5] Aspect A, Grangier P and Roger G 1982 *Phys. Rev. Lett.* **49** 91
[6] Aspect A, Dalibard J and Roger G 1982 *Phys. Rev. Lett.* **49** 1804
[7] Bennett C H, DiVincenzo D P, Smolin J and Wootters W K 1996 *Phys. Rev.* A **54** 3824
[8] Bennett C H, Brassard G, Popescu S, Schumacher B, Smolin J A and Wootters W K 1996 *Phys. Rev. Lett.* **76** 722
[9] Bennett C H, Bernstein H J, Popescu S and Schumacher B 1996 *Phys. Rev. Lett.* **53** 2046
[10] Vedral V and Plenio M B 1998 *Phys. Rev.* A **57** 1619
[11] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[12] Vidal G and Werner R F 2002 *Phys. Rev.* A **65** 032314
[13] Vedral V, Plenio M B, Rippin M A and Knight P L 1997 *Phys. Rev. Lett.* **78** 2275
[14] Henderson L and Vedral V 2001 *J. Phys. A: Math. Gen.* **34** 6899
[15] Terhal B, Horodecki M, Leung D and DiVincenzo D 2002 *J. Math. Phys.* **43** 4286
[16] Groisman B, Popescu S and Winter A 2005 *Phys. Rev.* A **72** 032317
[17] Hamieh S 2004 *Phys. Rev.* A **70** 052325
[18] Yurke B, McCall S L and Klauder J R 1986 *Phys. Rev.* A **33** 4033
[19] Campos R A, Saleh B E A and Teich M C 1989 *Phys. Rev.* A **40** 1371
[20] Leonhardt U 1993 *Phys. Rev.* A **48** 3265
[21] Horne M A, Shimony A and Zeilinger A 1989 *Phys. Rev. Lett.* **62** 2209
[22] Jaeger G, Horne M A and Shimony A 1993 *Phys. Rev.* A **48** 1023
[23] Jaeger G, Shimony A and Vaidman L 1995 *Phys. Rev.* A **51** 54
[24] Horodecki M, Horodecki P and Horodecki R 2000 *Phys. Rev. Lett.* **84** 2014
[25] Werner R F 1989 *Phys. Rev.* A **40** 4277
[26] Hamieh S, Qi J, Siminovitch D and Ali M K 2003 *Phys. Rev.* A **67** 014301
[27] Chan W L, Cao J P, Yang D and Gu S J 2007 *J. Phys. A: Math. Theor.* **40** 12143
[28] Kaszlikowski D, Sen(De) U, Vedral V and Winter A 2007 *Preprint* 0705.19.69